

LIFT CHARACTERIZATION OF AN AIRFOIL EMBEDDED WITH SPOILER AND FLAP-TIP IN WAKE VORTEX FLOWFIELD

Muyiwa Atoyebi^{1*}, Olawale Ismail², A.A. Dare³ and T.A.O. Salau⁴

^{1,2,3,4} Department of Mechanical Engineering, University of Ibadan, Ibadan, Nigeria

Corresponding author: atoyebimuyiwa@gmail.com

Phone: +234 (0) 803 422 0001

ABSTRACT: *Wingtip vortices are circular patterns of rotating air left behind a wing as it generates lift. Wingtip vortex minimizes lift, increases drag and increases fuel consumption. Therefore, this work involves lift characterization of an airfoil embedded with spoiler and flap-tip settings in wake vortex flow with an objective of alleviating wingtip wake vortices. The density of air was 1.15598 kg/m³ at altitude of 600 m. The velocity was 60 m/s while the dynamic viscosity was 0.0000177 N.s/m² at the same altitude. A Lifting Line Method (LLM) was developed and implemented in MATLAB. NACA 63-209 Airfoil was employed. The code termed Lifting Line Code (LLC) computes the lift coefficient during take-off of the aircraft. A wing of constant span of 16m, 5 different data of spoiler settings extensions and 25 different data of flap-tip extensions were numerically analyzed. For one data of spoiler setting, 5 different data of flap-tip were tested and the corresponding lift coefficients were recorded. The remaining data of both spoiler setting extension and flap-tip extensions were subsequently tested in the above order of combination and the values of lift coefficient were recorded as well. The values acquired from this analysis shows that the lift coefficient has increased by 2.5% for a wing with spoiler dimension of 16m by 2m (Aspect Ratio (AR) of 5.38); 3.07% for a wing with spoiler dimension of 18m by 2m (AR of 6.04); 2.46% for a wing with spoiler dimension 20m by 2m (AR of 6.69); 2.09% for a wing with 22m by 2m (AR of 7.35) and 1.82% for a wing with 24m by 2m (AR of 8.01). Hence it can be concluded that by increasing the span the lift coefficient increases and it can be further increased by adding wingtip devices (spoiler and flap-tip).*

Keywords: *Wingtip vortices, LLM, LLC, MATLAB, Spoiler setting, Flap-tip*

INTRODUCTION

1.1 Wake Vortex: Wake Vortex means the powerful rotating forces of air which is generated by the passage of an aircraft in flight. It is generated from the point when the nose landing gear of an aircraft leaves the ground on takeoff and ceases to be generated when the nose landing gear touches the ground during landing. Wake vortex includes vortices (wing-tip vortices and rotor vortices), thrust stream turbulence, jet blast, jet wash, propeller wash, and rotor wash both on the ground and in the air. A wake vortex encounter (WVE) is said to have occurred where another aircraft encounters such vortex. Wake vortex is created by lift and is a function of lift with an associated induced drag and not merely a creation of an aircraft being in the air [12]. Wake vortex study can be divided into near-wake field, extended near-wake field, mid-wake field and far-wake field. The near-wake region extends about 10-50 wingspans behind the generating aircraft and is composed of roll-up of multiple vortices off the wing surface, flaps, and tail. The extended near-wake field involves vortex roll-up and merging. The mid-wake field is made up of vortex drift-instabilities appearance. In the far-wake region, the multiple vortices combine into a counter-rotating vortex pair and the room temperature meteorological conditions dominate vortex transport and decay [26]. The study of trailing vortices is greatly significant to the development of novel wingtip devices which can provide an improved aerodynamic performance to the wing [28]. Wake vortex affects all airplanes of all sizes and shapes [25]. Vortices rotate around a core and are a few feet to several meters in diameter, depending on the size, configuration and speed of the aircraft. Vortices will spread laterally and descend behind and persist for a few minutes, sometimes longer in certain conditions. According to [15], the excitation of normal mode fluctuations results to coherent vast scale helical structures inside the vortical core. The radial development of these helical structures and the differential rotation result in the creation of a polarized vortex growth. [14] stated that the presence of trailing vortices formed behind lifting surfaces such as airplane wings and helicopter blades has great impacts for safety and performance.

1.2 Causes of Wake Vortex: Hazardous turbulence in the wake of an aircraft in flight is mainly caused by wing tip vortices. This type of turbulence is paramount because wing tip vortices decay quite slowly and can produce a vast rotational effect on an aircraft encountering them for several minutes after they have been generated.

1.3 Wake Vortex Formation: Every aircraft generates wake vortex from the moment the wing produces lift and the aircraft leaves the ground until it lands. This wake is also called wingtip or wake vortices. It forms when an aerofoil generates a pressure difference where the lowest pressure is over the upper surface and the highest pressure is under the wing. Figure 1 shows wake vortex formation. Figure 2 shows the structure of a roll-up vortex wake behind a crop duster through visualization enabled by smoke entrainment.

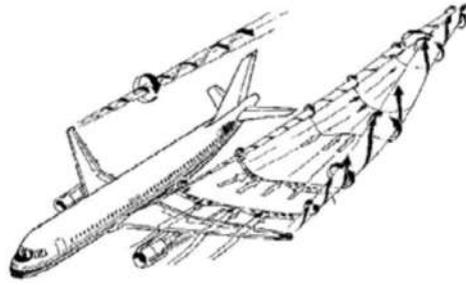


Figure 1: Wake Vortex Formation, Coustols [8]



Figure 2: The Structure of a Roll-up Vortex Wake behind a Crop Duster, [17]

1.4 Effects of Wake Vortex: These are felt in roll and yaw and are the most dangerous during takeoff and landing where there is no enough altitude to recover. The result of the wake on the aircraft depends on the wingspan and distance from the preceding aircraft. Sometimes only felt as rocking like flying through normal turbulence. In severe cases a total loss of control will be the result, recovery from this will depend on pilot skill, altitude, maneuverability and engine power. Wake vortex is influenced by the weight of the aircraft, speed, angle of attack and wing configuration. The conditions in the atmosphere have their effect how long the wake will persist [6]. Aircraft generated trailing vortices create a potential risk for following aircraft, owing to strong coherent flow structures [16]. In cruise altitudes the evolution of the aircraft's wake is vehemently dependent on the predominant meteorological conditions [24]. Additionally, in ground proximity the geometry and roughness of the ground surface affect the flow. The likelihood of encountering wake vortices increases largely during final approach in ground proximity since the vortices may not descend below the glide path, to leave the flight corridor vertically. Instead they rebound, owing to the interaction with the ground surface [20].

1.5 A Review of Modeling of Wake Vortices: Some aircraft require close proximity formation flying in order to reduce drag, ease air to-air refueling and combat effectiveness. A thorough understanding of the effects of wake vortex during wake vortex encounter is required. This is important in order to investigate safe and reliable operating procedures for close formation flight, analyze structural loading,

and design and test automatic control systems, to achieve this, a good and reliable models need to be developed [5]; [19]. There are two methods to the modeling of wake vortices for such purposes: (1) Look-up databases containing a priori values of the vortex effects obtained from either theoretical and/or experimental methods (CFD models, [23], [29], wind tunnel and/or flight test measurements Vlachos and [32]); (2) Online computational methods such as, from the simplest to the most involved: Prandtl's lifting line theory (single horseshoe vortex), e.g. references [27]; Vortex Lattice Methods (VLM) with or without viscous core, e.g. references [21]; improved methods taking account of the roll-up of the wake, e.g. references [33], and online CFD computations [22], [1]. The latter, obviously, requires enormous computational power. The first method has the advantage of accuracy, but is inflexible as it can only be used for a specified air-vehicle and a range of flight conditions. As for online computational methods, there is a clear trade-off between accuracy and rapidity of execution. [3]; [2] pioneered the field of modeling and investigation of aerodynamic coupling effects during air-to-air refueling using various computational methods, from relatively simple wake models based on a horseshoe vortex representation of the wing to more realistic roll-up models of the wake. They were followed by Blake and colleagues, i.e. [1]; [13], who investigated both theoretically and experimentally the effects of aerodynamic coupling during close proximity formation flying. In particular, they analyzed the optimum configuration for formation flight using a horseshoe vortex with viscous core and a vortex lattice method, and developed a simplified mathematical representation of the aerodynamic coupling for simulation of an arbitrary large number of tailless vehicles in close formation flight, using a combination of wind tunnel results and vortex lattice analysis. The benefits of formation flying in terms of induced drag for the trailing aircraft were later confirmed by the NASA Dryden Flight Research Center Autonomous Formation Flight programme (AFF), where flight tests demonstrated up to 18% reduction in fuel consumption, [18]; [31]. Finally, Dogan, Venkataramanan and Blake i.e. [13], developed a method to calculate aerodynamic coupling between aircraft flying in close proximity within dynamic simulations without explicitly computing the additional force and moment coefficients induced by the leading vehicle on the trailing aircraft: they approximated the non-uniform induced velocity field as uniform wind components and gradients, and used them directly in the equations of motion of the trailing vehicle(s) with wind terms. This paper describes the development of a lifting line method and mathematical models that incorporate wake vortex effects associated with Sikorsky aircraft vehicles flying in close proximity. The following sections are organized as follows. Section 1 contains introduction to wake vortex and a review modeling of wake vortices; Section 2 describes the theoretical development of the wake vortex model and the methodology; Section 3 presents the results and finally, in Section 4, conclusions are drawn.

2. METHODOLOGY

2.1 Lifting Line Theory (LLT): In the wake vortex analysis, it is necessary that the lift force that a wing is generating must first be calculated. Then, with the aid of wing parameters, the induced rolling moment can be calculated as well. The LLT technique essentially comes from the area of Aerodynamics. The LLT technique allows the determination of the amount of lift force that is generated by a wing. Meanwhile, all wing data such as wing area, airfoil section and its features, aspect ratio, taper ratio, wing incidence, and high lift device type and data must be made available. By solving aerodynamic equations (Lifting Line Equation) simultaneously, the amount of lift that a wing is producing can be determined.

2.2 Lifting Line Equation: The following group of equation was solved to find A_1 to A_n :

$$\mu (\alpha_0 - \alpha) = \sum_{n=1}^N A_n \sin(n\theta) \left(1 + \frac{\mu n}{\sin(\theta)} \right)$$
 This equation is the heart of the theory and is referred to as the *lifting-line* equation or *monoplane* equation. The equation initially developed by Prandtl. In this equation, N denotes number of segments; α denotes segment's angle of attack; α_0 denotes segment's zero-lift angle of attack; and coefficients A_n are the intermediate unknowns. The parameter μ is defined as follows:

$\mu = \frac{\bar{c}_i C_{l\alpha}}{4b}$ where \bar{c}_i denotes the segment's mean geometric chord, $C_{l\alpha}$ denotes segment's lift curve slope in 1/rad; and "b" is the wing span. If the wing has a twist (α_0), the twist angle must be applied to all segments linearly. Thus, the angle of attack for each segment is reduced by deducting the corresponding twist angle from wing setting angle. If the theory is applied to a wing in a take-off operation, where flap is deflected, the inboard segments have larger zero-lift angle of attack (α_0) than outboard segments. Each segment's lift coefficient is determined using the following equation:

$C_L = \frac{4b}{c_i} \sum A_n \sin(n\theta)$, the variation of segment's lift coefficient (CL) versus semispan (i.e. lift distribution) is plotted. Wing total lift coefficient is determined using the following equation: $C_{L_w} = \pi \cdot AR \cdot A_1$, where AR is the wing aspect ratio.

2.3 Analysis of Add-on Devices

In this research work, a wing of constant aspect ratio, wing area and wing span is analyzed under ideal flow conditions for reduction in induced drag. A code in Matlab is used for analysis. Vortex lattice method is used in the code to solve the equations in the in viscid flow conditions assumed. A rectangular wing with Spoiler, Wingtip/Flap tip and a combination of spoiler and Flap tip, are studied. Using mathematical formulas, a numerical analysis is carried out to study the trailing vortices and the induced drag produced due to the wingtip devices. The matlab code includes data and analysis capabilities based on lifting line theory (LLT) and the vortex lattice method (VLM). This study is carried out using the vortex lattice method (VLM) on the mean camber line. NACA 63-209 airfoil is used. The free stream velocity, wing span and angle of attack considered are constant throughout this study. The matlab code calculates the lift coefficient, drag coefficient, aerodynamic efficiency and rolling moment. **Spoilers:** Spoilers are plates on the top surface of a wing that can be extended upward into the airflow to spoil it. The modification was done to the spoiler by creating two slots (2 slots). **Wing-tip or Flat-tip:** This is a device intended to improve the efficiency of fixed-wing aircraft. Wingtip/Flat-tip increases the lift generated at the wingtip by smoothing the airflow across the upper wing near the tip and reduces the lift-induced drag caused by wingtip vortices, improving lift-to-drag ratio. A rectangular wing (Sikorsky's aircraft wing) of constant span of 16m, 5 different data of spoiler settings extensions and 25 different data of flap-tip extensions were numerically analyzed. For one data of spoiler setting, 5 different data of flap-tip were tested and the corresponding lift coefficients were recorded. The remaining data of both spoiler setting extension and flap-tip extensions were subsequently tested in the above order of combination and the values of lift coefficient recorded as well. Altitude of 600m; Density, $\rho = 1.15598 \text{ kg/m}^3$; and Dynamic Viscosity, $\mu = 0.0000177 \text{ N.s/m}^2$, Flight Altitudes, [11].

2.4 A Short Introduction to MATLAB

The name MATLAB stands for MATrix LABoratory. MATLAB is a special purpose computer program optimized to perform scientific and engineering calculations. It began as a program designed to perform matrix mathematics but over the years, it has grown into a flexible computing system capable of solving essentially any technical problem [7]; [4]; [6]. All calculations in this paper were performed with MATLAB.

3. RESULTS AND DISCUSSION

Table 3: Spoiler and Flap-tip Settings

| Wing type | Wing with Spoiler A | Wing with both spoiler and Flap-tip A1 | Wing with both spoiler and Flap-tip A2 | Wing with both spoiler and Flap-tip A3 | Wing with both spoiler and Flap-tip A4 | Wing with both spoiler and Flap-tip A5 |
|------------------------------------|---------------------|--|--|--|--|--|
| Span (m) | 16.00 | 16.40 | 16.80 | 17.20 | 17.60 | 18.00 |
| Wing area (m ²) | 48.77 | 49.98 | 51.21 | 52.43 | 53.64 | 54.86 |
| Aspect ratio | 5.25 | 5.38 | 5.51 | 5.64 | 5.77 | 5.91 |
| Lift Coefficient (C _L) | 0.23928 | 0.2442 | 0.2458 | 0.2473 | 0.2477 | 0.2503 |

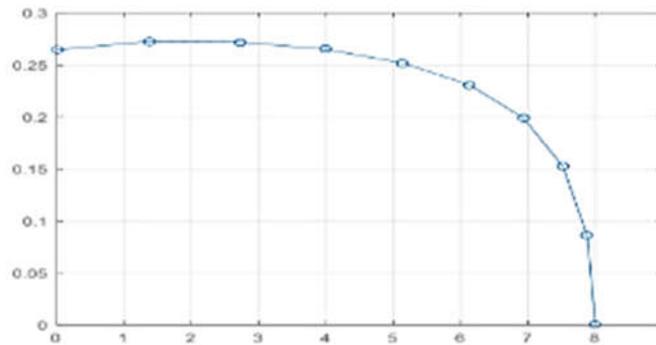


Figure 3: Title ('Lift distribution'); xlabel ('Semi-span location (m)'); ylabel ('Lift coefficient'); spoiler setting = 16m, AR = 5.25; A (1) = 48.77 m², C_L = 0.23928

Table 4: Spoiler and Flap-tip Settings

| Wing type | Wing with Spoiler B | Wing with both spoiler and Flap-tip B1 | Wing with both spoiler and Flap-tip B2 | Wing with both spoiler and Flap-tip B3 | Wing with both spoiler and Flap-tip B4 | Wing with both spoiler and Flap-tip B5 |
|------------------------------------|---------------------|--|--|--|--|--|
| Span (m) | 18.00 | 18.40 | 18.80 | 19.20 | 19.60 | 20.00 |
| Wing area (m ²) | 54.86 | 56.08 | 57.30 | 58.52 | 59.74 | 60.96 |
| Aspect ratio | 5.91 | 6.04 | 6.17 | 6.30 | 6.43 | 6.56 |
| Lift Coefficient (C _L) | 0.2503 | 0.2517 | 0.2530 | 0.2543 | 0.2556 | 0.2567 |

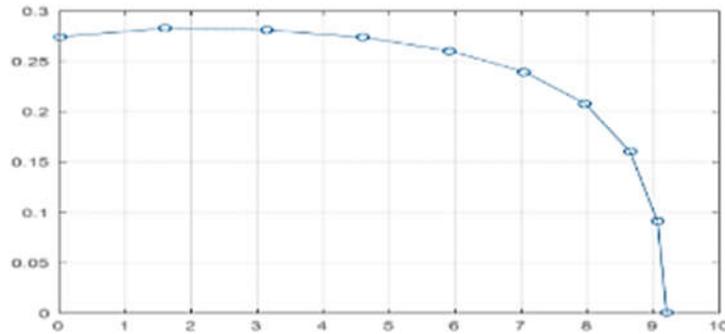


Figure 4: Title ('Lift distribution'); xlabel ('Semi-span location (m)'); ylabel ('Lift coefficient'); spoiler and flap-tip setting = 18.4m, AR = 6.04; A (1) = 56.08 m², C_L = 0.2517

3.2 Discussion of Results: The values acquired from this analysis shows that the lift coefficient has increased by 2.5% for a wing with spoiler dimension of 16 m by 2 m (AR of 5.38); 3.07% for a wing with spoiler dimension of 18 m by 2 m (AR of 6.04); 2.46% for a wing with spoiler dimension 20 m by 2 m (AR of 6.69); 2.09% for a wing with 22m by 2m (AR of 7.35) and 1.82% for a wing with 24 m by 2 m (AR of 8.01). With the combination of spoiler and flap tip, there is a reduction of peak vorticity, a reduction of maximum cross-flow vorticity, an enlargement of vortex core, spread of vorticity/distribution of vorticity but circulation strength flap vortex remains unchanged. The selected devices re-distributed and interacted with the vorticity because of the concentration of the vorticity near the wing tip. The selected near-field device change the vortex width and rolling momentum at the end of the diffusion regime. The wake vortex break up, is due to long-wave instabilities.

4. Conclusion: The analysis carried out using Lifting Line Theory (LLT), Vortex Lattice Method (VLM) and numerical calculations in Matlab generated reasonable results. The effects of wingtip devices were revealed, which shows significant contribution to alleviate wingtip vortices. The values acquired from this analysis shows that the lift coefficient has increased by 2.5% for a wing with spoiler dimension of 16m by 2m (AR of 5.38); 3.07% for a wing with spoiler dimension of 18m by 2m (AR of 6.04); 2.46% for a wing with spoiler dimension 20m by 2m (AR of 6.69); 2.09% for a wing with 22m by 2m (AR of 7.35) and 1.82% for a wing with 24m by 2m (AR of 8.01). Hence it can be concluded that by increasing the span the lift coefficient increases and it can be further increased by adding a wingtip devices. These wingtip devices prevent the formation of wingtip vortices by redirecting the flow of fluid emanating from the lower surface of the wing. From the results obtained, using a wingtip device (A combination of spoiler setting and flap-tip) has proved efficient, in reducing the induced drag but its limitations of increased structural weight and aero elasticity can be prevented through careful selection and design.

REFERENCES

- [1] Blake, W. and Gingras, D. (2004). "Comparison of Predicted and Measured Formation Flight Interference Effects," *Journal of Aircraft*, Vol. 41, No. 2, Mar.-Apr. 2004, pp. 201–207.
- [2] Bloy, A. and Joumaa, M. (1995). "Lateral and directional stability and control in air-to-air refuelling," *Proceedings of the Institution of Mechanical Engineers Part G-Journal of Aerospace Engineering*, Vol. 209, No. G4, 1995, pp. 299–305.
- [3] Bloy, A. and Lea, K. (1995). "Directional Stability of a Large Receiver Aircraft in Air-to-Air Refueling," *Journal of Aircraft*, Vol. 32, No. 2, Mar.-Apr. 1995, pp. 453–455.
- [4] Brian, D.H. (1997). *Essential MATLAB for Scientists and Engineers*, 1st edition, Arnold publisher, 338 Euston Road, London.
- [5] Capetta, R., Giulietti, F. and Innocenti, M. (2001). "WakeCAD: Aerodynamics Interference Calculation Toolbox for Aircraft De-sign, Simulation and Control," *AIAA Guidance, Navigation, and Control Conference and Exhibit*, AIAA 2001-4180, Montreal, Canada, Aug. 6-9 2001.

- [6] Chao, L. (2007). *'Wake Vortex Encounter Analysis with Different Wake Vortex Models Using Vortex-Lattice Method, A numerical study'*. Master of Science Thesis in Aerospace Engineering, Faculty of Aerospace Engineering, Delft University of Technology. <https://www.tudelft.nl/en/>
- [7] Chapman, S.J. (2000). *MATLAB Programming for Engineers*, Brooks Cole Publishing Company, 'www.brookscole.com', USA.
- [8] Coustols, E. (2005). *'An Overview of European Projects on Wake Vortices'* in connection to Working Group 7 activities', Principles of Wake Vortex alleviation. ONERA/DMAE, Toulouse-France.
- [9] Deborah, S. and James, F.W. (2009). *'Modeling of Wake Vortex Effects for Unmanned Air Vehicle Simulations'*. Dynamics, Simulation and Control Group, Department of Aerospace Sciences, Cranfield University, Cranfield, Bedfordshire, MK43 0AL, UK, AIAA, Modeling and Simulation Technologies Conference 10 - 13 August 2009, Chicago, Illinois, AIAA 2009-5686.
- [10] Deborah, S., James F. W. and Alastair, C. (2009). *'Simulation of Wake Vortex Effects for UAVs in Close Formation Flight'*. Aeronautical Journal, volume 113, Issue 1149, pp 727-738, Department of Aerospace Sciences Cranfield University, Cranfield, UK.
- [11] de Bruin, A. and Winkelmanns., G. (2005). *'Cross-flow kinetic energy and core size growth of analytically defined wake vortex pairs'*. Technical Papers. NLR-CR-2005-412.
- [12] Dinesh, M., Premkumar, P. S., Rao, J.S. and Senthilkumar. C. (2013). *'Performance Analysis of Winglets Using CFD Simulation'*. Altair Technology Conference, Department of Aero. Engg., Kumaraguru College of Technology (KCT), Chinnavedampatti, Coimbatore – 641049, India.
- [13] Dogan, A., Venkataramanan, S. and Blake, W. (2005). *"Modeling of Aerodynamic Coupling Between Aircraft in Close Proximity,"* Journal of Aircraft, Vol. 42, No. 4, Jul.-Aug. 2005, pp. 941–955.
- [14] Duraisamy, K. and Lele, S. K. (2006). *'DNS of Temporal Evolution of Isolated Turbulent Vortices'*. Center for Turbulence Research, Proceedings of the Summer Program, Aerospace Engineering, University of Maryland, College Park.
- [15] Duraisamy, K. and Lele S.K. (2008). *'Evolution of Isolated Turbulent Trailing Vortices'*. American Institute of Physics. DOI: 10.1063/1.2840200.
- [16] Gerard, V.M. (2000). *Data Analysis in the Earth Sciences using MATLAB*, Prentice Hall, Upper Saddle River, New Jersey 07458.
- Gerz, T., Holz"apfel, F. Darracq, D. (2002). *Commercial Aircraft Wake Vortices*. Progress in Aerospace Sciences, Vol. 38, No. 3, PP. 181–208.
- [17] Götz, B. (2006). *'A Higher Order Vortex-Lattice Method with a Force-Free Wake'*. A Ph.D Thesis in Aerospace Engineering, the Graduate School College of Engineering, The Pennsylvania State University. <https://www.upenn.edu>
- [18] Hansen, J. and Cobleigh, B. (2002). *"Induced Moment Effects of Formation Flight Using two F/A-18 Aircraft,"* AIAA Atmospheric Flight Mechanics Conference and Exhibit , AIAA 2002-4489, Monterey, CA, Aug. 5-8 2002.
- [19] Holzapfel, F., Frech, M., Gerz, T., Tafferner, A., Hahn, K.-U., Schwarz, C., Joos, H.-D., Korn, B., Lenz, H., Luckner, R. and Hohne, G. (2008). *"Aircraft wake vortex scenarios simulation package – WakeScene,"* Aerospace Science and Technology, 2008.
- [20] Holz"apfel, F. and Steen, M. (2007), *Aircraft Wake-Vortex Evolution in Ground Proximity: Analysis and Parametrization*. AIAA Journal, Vol. 45, PP. 218–227.
- [21] Karkehabadi, R. (2004). *"Wind-Tunnel Measurements of Hazard Posed by Lift-Generated Wakes,"* Journal of Aircraft, Vol. 41, No. 6, Nov.-Dec. 2004, pp. 1424–1429.
- [22] Kenny, J., Takeda, K. and Thomas, G. (2008). *"Real-time computational fluid dynamics for flight simulation,"* The Interservice/Industry Training, Simulation & Education Conference (IITSEC), Orlando, FL, Dec. 01-04 2008.
- [23] Le Moigne, A. and Qin, N. (2006). *"LES with numerical dissipation for aircraft wake vortices,"* 44th AIAA Aerospace Sciences Meeting and Exhibit , AIAA 2006-1258, Reno, NV, Jan. 9-12 2006.
- [24] Misaka, T., Holz"apfel, F., Hennemann, I., Gerz, T., Manhart, M. and Schwertfirm, F. (2012). *Vortex Bursting and Tracer Transport of a Counter-rotating Vortex Pair*. Physics of Fluids, Vol. 24, PP. (025104–1) – (025104–21).

- [25] Momchil, D., (2012). '*Experimental and Numerical Study on Wingtip Mounted Propellers for Low Aspect Ratio UAV Design*'. A Master of Science Thesis in Aerospace Engineering, Delft University of Technology. <https://www.tudelft.nl/en/>
- [26] Nash'at N. A., Fred H. P. and Brad P.R. (2013). '*Numerical Simulation of the Aircraft Wake Vortex Flowfield*'. NASA Langley Research Center, Hampton, Virginia, 23681, American Institute of Aeronautics and Astronautics.
- [27] Pachter, M., D'Azzo, J. and Proud, A. (2001). "*Tight Formation Flight Control*," Journal of Guidance, Control, and Dynamics, Vol. 24, No. 2, Mar.-Apr. 2001, pp. 246–254.
- [28] Revelly, A., Iaccarino, G. and Wu, X. (2006). '*Advanced RANS modeling of wingtip vortex flows*'. Center for Turbulence Research Proceedings of the Summer Program, MACE, University of Manchester, UK.
- [29] Spence, G., Le Moigne, A., Allerton, D. and Qin, N. (2007). "*Wake Vortex Model for Real-Time Flight Simulation Based on Large Eddy Simulation*," Journal of Aircraft, Vol. 44, No. 2, Mar.-Apr. 2007, pp. 467–475.
- [30] Tomas, M. (2000). '*A Vortex Lattice MATLAB Implementation for Linear Aerodynamic Wing Applications*'. Master Thesis in Aeronautics Engineering, Royal Institute of Technology (KTH). Department of Aeronautics. <https://www.kth.se>
- [31] Vachon, M., Ray, R., Walsh, K. and Ennix, K. (2002). "*F/A-18 Aircraft Performance Benefits Measured During the Autonomous Formation Flight Project*," AIAA Flight Mechanics Conference and Exhibit , AIAA 2002-4491, Monterey, CA, Aug. 5-8 2002.
- [32] Vlachos, P. and Telionis, D. (2003). "*Wing-tip-to-wing-tip Aerodynamic Interference*," 41st Aerospace Sciences Meeting and Exhibit, Reno, NV, 2003.
- [33] Wang, Z. and Mook, D. (2003). "*Numerical Aerodynamic Analysis of Formation Flight*," 41st Aerospace Sciences Meeting and Exhibit, AAIA 2003-610, Reno, NV, Jan. 6-9 2003.
- [34] de Bruin, A. & Winckelmans., G. (2005). Cross-flow kinetic energy and core size growth of analytically defined wake vortex pairs. Technical Papers. NLR-CR-2005-412.