

A SPEED-DEPENDENT DISCRETIZATION FOR FASTER AND MORE ACCURATE MILLING STABILITY ANALYSIS

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Abstract: As the major aim, a speed-dependent discretization method which utilizes the approximation parameter as a function of rotational speed of the spindle is proposed. Brief details of stability condition of regenerative chatter of end-milling is presented. The linear least squares approximated full-discretization method (FDM) is used as a reference method against which the accuracy and computational time (CT) of the proposed method is judged. The method is verified for one-degree of freedom (1DOF) milling to improve the accuracy of the low-speed domain while simultaneously providing enormous savings in CT. The classical approach of using fixed approximation parameter in the FDM, and other methods that include the semi-discretization, is noted for biased filtration of computational inaccuracy to concentrate in the low-speed domain. The effort by this approach to provide high accuracy at low speed by increasing the approximation parameter causes an intolerable rise in CT. The proposed method solves this problem within relatively short CT while providing a fairer distribution of accuracy that favours the low-speed domain thus giving stability diagrams that are reliable in all speed range.

Keywords: Milling process; full-discretization method, Chatter, Stability analysis

1. INTRODUCTION

Need for stability analysis of delayed periodic differential equations is always encountered by engineers interested in machine tool vibrations [1-18], robotics control [19] and stabilization [20-22]. Regenerative vibration of milling tools is very popular as a member of the set of phenomena governed by such equations. Parametric excitation in milling stems from periodic variation in angular position of the cutting edges while the delay results from effect profile of a tooth pass has on that of the succeeding tooth pass. The effect of the delay is to cause the system to have infinite dimensional phase space. The stability of the system is dependent on infinitely many eigen-values of the system's infinite dimensional monodromy operator. Finite approximation of the infinite monodromy operator is needed for stability characterization of milling process. A number of methods have been established for this purpose. Amongst the methods are time domain simulations of chatter [1], a method that utilizes the mean of the Fourier series of the dynamic milling coefficients called the Zeroth Order Approximation (ZOA) method [2, 3], semi-discretization [4 - 7], Chebyshev Polynomials [8], temporal finite element [5, 8, 9] and full-discretization method [10, 11, 15-18]. The aim in this work is to explore ways of shortening CT and improving accuracy of milling stability charts. Computational illustration will be based the full-discretization method (FDM) though the proposal is aimed to be applicable across board. The FDM is already known to be much more time conserving than the semi-discretization method [10]. The FDMs based on interpolation theory and those based on approximation theory (least squares method) are of same accuracy but the latter are superior for producing monodromy matrices that are generated with less number of numerical calculations thus saving CT [18]. For this reason, this work is based on the least squares approximated FDM. The approach in this work which is not yet mentioned in literature is the use of speed-dependent approximation parameter. By this, the number of calculation points is made to be dependent on the rotational velocity. A number of equations that should govern the variation of approximation parameter during computation of stability lobes are proposed.

2. MATERIALS AND METHODOLOGY

2.1. Mathematical model

A well-known mathematical model of regenerative chatter of 1DOF (compliant in only the feed direction) milling tool when cutting a rigid workpiece is given in equation (1) [12, 4, 10, 11, 15-18, 25].

$$\dot{q}(t) = Aq(t) + B(t)q(t) - B(t)q(t-\tau) \quad (1)$$

The vector $q(t)$ is the milling state. The matrices A and $B(t)$ are respectively the constant matrix that contains the transient response of the system and the periodic coefficient matrix that captures the periodicity of cutting force of the forced milling process

$$A = \begin{bmatrix} 0 & 1 \\ -\omega_n^2 & -2\xi\omega_n \end{bmatrix} \quad (2)$$

$$B(t) = \begin{bmatrix} 0 & 0 \\ -\frac{wh(t)}{m} & 0 \end{bmatrix} \quad (3)$$

The natural frequency and damping ratio of the tool are given in terms of the tool modal parameters as $\omega_n = \sqrt{k/m}$ and $\xi = c/2\sqrt{mk}$. The non-linear cutting force model on j th tooth used in arriving at equation (1) is

$$F_{t,j}(t) = C_t w [f_a \sin\theta_j(t)]^\gamma \quad (4)$$

$$F_{n,j}(t) = C_n w [f_a \sin\theta_j(t)]^\gamma = X F_{t,j}(t)$$

where w is the axial depth of cut, C_t and C_n are the tangential and normal cutting coefficients associated with the workpiece, X is the ratio C_n/C_t , f_a is the dynamic feed per tooth (ie. feed within single discrete delay $\tau = 60/(N\Omega)$) and γ is a positive exponent. The τ -periodic function $h(t)$ called the specific force variation and other details of the model can be specifically found in [18, 25, 26]. The equivalent model for 2DOF tool (compliant in both the feed and feed-normal directions) is specifically derived and described in [25].

2.2. Chatter Discrete Mapping via Full-Discretization Method

The method of full-discretization for study of milling stability is based on equation (1). The principal period which is same as the discrete delay τ for milling tool with uniform pitch is discretized into equal time intervals Δt . The discrete interval is given as $\Delta t = \tau/r = t_{i+1} - t_i$ where $i = 0, 1, 2, \dots (r-1)$, $t_i = i\tau/r = i\Delta t = i(t_{i+1} - t_i)$ and the discretization integer r is called the approximation parameter. Equation (1) is represented as

$$\dot{y}(t) = Ay(t) + B(t)y(t) - B(t)y(t-\tau) \quad (5)$$

in the discrete interval $[t_i, t_{i+1}]$ and solved using direct integration scheme to give

$$y_{i+1} = e^{A\Delta t} y_i + \int_{t_i}^{t_{i+1}} e^{A(t_{i+1}-s)} [B(s)y(s) - B(s)y(s-\tau)] ds \quad (6)$$

The temporal functions $y(s)$, $y(s-\tau)$ and $B(s)$ are approximated by linear Lagrange interpolation polynomial in [10]. The least squares approximated FDM presented in the work [18] is the very time conserving state-of-art FDM. The least squares approximated FDM approximates the state term $y(s)$ as

$$y(s) = [a(s)]^T \left\{ \sum_{l=i-p+1}^{i+1} a(s_l) [a(s_l)]^T \right\}^{-1} \sum_{l=i-p+1}^{i+1} a(s_l) y_l \quad (7)$$

where $a(s) = \{s^0 \ s^1 \ s^2 \ \dots \ s^p\}^T$ is called the polynomial of basis vector of p -th order and the summation is considered applicable to each of the elements of the matrix $a(s_l) [a(s_l)]^T$ and vector $a(s_l) y_l$. Substituting $p = 1$ gives

$$y(s) = \frac{1}{\Delta t} (\Delta t - s) y_i + \frac{1}{\Delta t} s y_{i+1} \quad (8)$$

Similarly the delayed state $y(s-\tau)$ and periodic coefficient matrix $B(t)$ for which p is limited to unity read

$$y(s-\tau) = \frac{1}{\Delta t} (\Delta t - s) y_{i-r} + \frac{1}{\Delta t} s y_{i+1-r} \quad (9)$$

$$B(s) = \frac{1}{\Delta t} (\Delta t - s) B_i + \frac{1}{\Delta t} s B_{i+1} \quad (10)$$

Equations (8)-(10) are substituted in equation (6) and solved as elaborated in [18] to give the linear least squares approximated FDM with the monodromy operator

$$\psi = M_{r-1} M_{r-2} \dots \dots M_0 \quad (11)$$

where exact description of ψ can be found in [18].

The monodromy ψ is a linear operator that transforms the delayed state x_0 to the present state x_{r+1} .

Asymptotic stability of equation (1) requires all characteristic exponents to have negative real parts. The criterion for asymptotic stability of equation (1) can also be stated to mean that all characteristic multipliers have modulus less than one. This means that stability of the system requires all characteristic multipliers to exist inside a unit circle centred at the origin of the complex plane. At critical operating conditions when there exist characteristic multipliers such that $|\mu_i| = 1$, the corresponding characteristic exponents are pure imaginary. This is the basic idea of tracking the stability transition curves of periodic differential equations.

3. RESULTS AND DISCUSSION

Before proceeding with numerical computations of stability lobes, the specifications of the personal computer used in computation are as follows: manufacturer; Hewlett-Packard, Model; Presario CQ61 notebook PC, Rating; 3.9 (Windows Experience Index), Processor; AMD Sempron(tm) M120 2.10GHz, Installed Memory (RAM); 2.00GB (1.75GB usable), System type; 64-bit Operating system. The MATLAB 7.5.0 (R2007b) programs used are written with loops for specific periodic cutting force variation $h(t)$ for flexibility that allows for computation for any number

of cutting edges. Even though avoiding loops hastens computations, writing $h(t)$ out for different tools with different cutting edges could introduce errors.

3.1. Computation of stability diagrams using fixed approximation parameter

In what follows the size of integration interval $[t_i, t_{i+1}]$ is designated α instead of Δt . When computation of stability diagrams is done at constant approximation parameter r and α will vary with spindle speed according to the equation

$$\alpha = \frac{\tau}{r} = \frac{60}{N\Omega r} \quad (12)$$

Normally, computations for stability diagrams are done on a regular $l \times n$ data grid of the plane of spindle speed and depth of cut. This means that the size of α at the j th ($j = 1, 2, \dots, l + 1$) computation step of the speed axis is

$$\alpha_j = \frac{60}{N\Omega_j r} = \frac{60}{Nr[\Omega_1 + (j-1)\Delta\Omega]} \quad (13)$$

where $\Delta\Omega$ is the spindle speed step. It is obvious from equation (13) that α falls continuously with rise of j from 1 to $l + 1$. Normally accuracy of stability diagrams increases with decrease in α but a point is reached where improvement in accuracy becomes too marginal to be of any use. At this point decrease in α should be halted to avoid time-wasting computational details. This kind of computational deficiency is not yet addressed and becomes the strong point of novelty of this work.

3.2. Computation of stability diagrams using speed-dependent approximation parameter

The characteristic multipliers of the system are only computed through numerical means thus there is no analytical way to determining the value of α that gives them within a specified error bound. The only way to ensure convergence, hence accuracy, is decreasing α but this decrease should not be indefinite as already pointed out. If, generally speaking, there are f integration intervals in a natural frequency period then α becomes

$$\alpha = \frac{2\pi}{\omega_n f} \quad (14)$$

The value of f that ensures minimum accuracy is dependent on ω_n in a way that f is smaller when ω_n is larger. The approximation parameter generally becomes

$$r = \text{ceil}\left(\frac{\tau}{\alpha}\right) = \text{ceil}\left(\frac{30\omega_n f}{N\Omega\pi}\right) \quad (15)$$

Where ceil is a function that is used to ensure that r is an integer greater than zero by rounding the value of $30\omega_n f/(N\Omega\pi)$ to the nearest integer greater than or equal to $30\omega_n f/(N\Omega\pi)$.

The first case of use of (15), which is the traditional case, involves keeping r constant by keeping f constant and Ω constant at Ω_1 . This traditional case is frequently implemented with $f = 20$ and $\Omega = \Omega_1 = 750\text{rpm}$ such that $r = \text{ceil}[30 \times 920.49 \times 20 / (4 \times 750 \times \pi)] = \text{ceil}(58.60021343) = 59$ in computing stability diagrams of four-tooth miller on a 200 by 100 data grid as shown in Figures 1. The parameters used for computation are as seen in Table 1. The second case of use of equation (15) involves keeping α constant by keeping f constant such that r falls with rise in Ω . The spindle speed Ω contained in equation (15) is made to be the same as the spindle speed at every step of computation. Since the time needed to calculate ψ is heavily dependent on number of integration intervals which is same as r , computational time at steps of spindle speed falls fast with rise in Ω . This is expected to accrue a considerable gain in speed of computation by avoiding time-wasting computational details pointed out earlier. Trials using $f = 20$ showed that the stability curve is compromised at higher-speed domain though a 90 percentage gain in CT is achieved.

This kind of slight error generally gets less noticeable with rise in radial immersion and number of cutting edges. The inaccuracy may be of marginal consequence in technological application.

Another approach is varying f such that higher values are used in the higher-speed portion of stability computation. This informs the third case of considering f as a function of speed Ω of form

$$f = s\Omega^\beta + f_0, \quad \beta > 0 \quad (16)$$

Where s and f_0 are respectively given in terms of the starting speed Ω_1 , ending speed Ω_{l+1} , starting factor f_1 and ending factor f_{l+1} as

$$s = \frac{f_{l+1} - f_1}{\Omega_{l+1}^\beta - \Omega_1^\beta} \quad (17)$$

$$f_0 = \frac{f_1 \Omega_{l+1}^\beta - f_{l+1} \Omega_1^\beta}{\Omega_{l+1}^\beta - \Omega_1^\beta} \quad (18)$$

The aim in formulating equation (16) is to achieve two things when f varies between the limits $[f_1, f_{l+1}]$ due to variation of Ω between the limits $[\Omega_1, \Omega_{l+1}]$. The two things are; savings in CT relative to the first case in which r and f are fixed and improvement in accuracy relative to the second case in which only f is fixed. When β lies in the range $0 < \beta < 1$ the rate of change of f with Ω within the limits $[f_1, f_{l+1}]$ given by $df/d\Omega = \beta s \Omega^{\beta-1}$ falls as Ω rises

within the limits $[\Omega_1, \Omega_{l+1}]$. When β lies in the range $\beta > 1$ the rate of change of f with Ω within the given limits rises as Ω rises. For the boundary case when $\beta = 1$ the rate of change of f with Ω will be fixed at s . Since variations are within fixed limits, curves of f with higher β will lie below those with lower β . This means that the average value of f given by equation (19) rises with fall in β

$$f_{\text{mean}} = \frac{1}{\Omega_{l+1} - \Omega_1} \int_{\Omega_1}^{\Omega_{l+1}} f d\Omega = \frac{1}{\beta+1} \left(\frac{\Omega_{l+1}^{\beta+1} - \Omega_1^{\beta+1}}{\Omega_{l+1} - \Omega_1} \right) + f_0 \quad (19)$$

The meaning of the above is that fall in β will cause rise in CT but will achieve a more even accuracy (that is accuracy spread over the whole range of spindle speed) in the resulting stability diagram. That notwithstanding, the range $\beta > 1$ is recommended for faster computation and acceptable accuracy. The graph of f at $\beta = 0.1, 0.25, 0.5, 1, 2, 4$ and 10 for f varying between the limits $[20, 80]$ due to variation of Ω between the limits $[750, 6000]$ rpm is shown Figure 2.

Equations (16), (17) and (18) are used in (15) to give the value of r at each spindle speed Ω as

$$r = \text{ceil} \left\{ \frac{30\omega_n}{N\Omega\pi(\Omega_{l+1}^\beta - \Omega_1^\beta)} \left[f_{l+1} (\Omega^\beta - \Omega_1^\beta) + f_1 (\Omega_{l+1}^\beta - \Omega^\beta) \right] \right\} \quad (20)$$

In light of Figure 2, the recommended values for β reads

$$\beta = N \quad (21)$$

where N is the number of cutting edges. The stability curve is re-computed with r as given in Equation (20) for f_1 fixed at 20 and f_{l+1} having the values 130 and 200 for $l = 200$. The results are shown as Figures 3 and 4, respectively. Comparing each of Figures 3 and 4 with Figure 1 gives that this speed-dependent algorithm gives reliable results. The final factor f_{l+1} should be large enough to ensure accuracy. It is seen that each of $f_{l+1} = 200$ sufficiently computes the stability curve with 62.916 percentage gain in CT.

Table 1. Parameters of milling process sourced from [8]

No	Symbol	Value
1	ω_n	$920.49 \text{ rad s}^{-1}$
2	ξ	0.0032
3	C_t	$5.5 \times 10^8 \text{ Nm}^{-1-\gamma}$
4	C_n	$2 \times 10^8 \text{ Nm}^{-1-\gamma}$
5	X	0.3636
6	m	2.5729 kg
7	γ	1

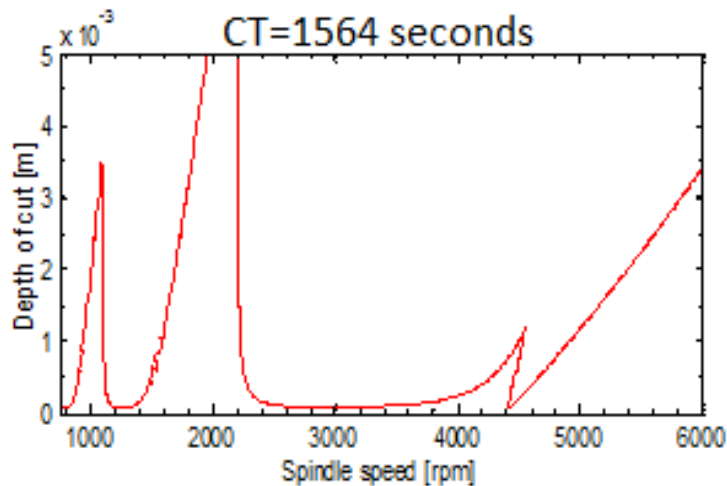


Fig. 1. Stability boundary curves of four-tooth miller computed using equation (15) when (a) $\rho = 0.25$, $f = 20$ and Ω fixed at $\Omega_1 = 750 \text{ rpm}$ such that $r = 59$.

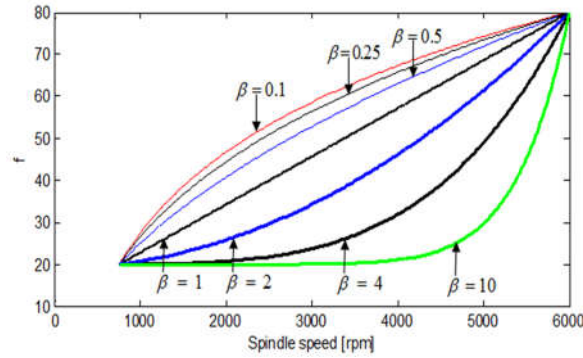


Fig. 2. (a) Variation of factor f with Ω at different values of β

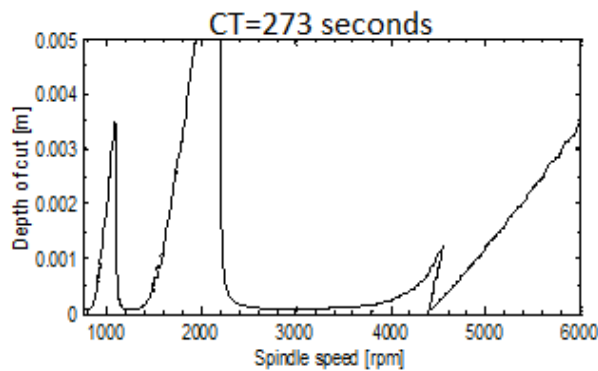


Fig. 3. Stability boundary curves of four-tooth miller at $\rho = 0.25$ computed using equation (20) with $f_1 = 20$ and $f_{1+1} = 130$

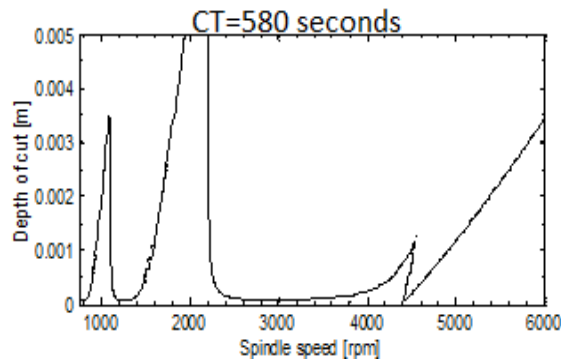


Fig. 4. Stability boundary curves of four-tooth miller at $\rho = 0.25$ computed using equation (20) with $f_1 = 20$ and $f_{1+1} = 200$

4. CONCLUSION

This work introduced a time-saving and accuracy improving speed-dependent FDM for computing stability lobes. The presented method is based on some proposed speed-dependent algorithms for the discretization of system period. The traditional approach of discretizing all the delays of spindle speed axis into equal number of intervals has the shortcoming that inaccuracy of computed stability diagrams is concentrated at low speed and shades into better accuracy with rise in speed. A proposed algorithm based on keeping f constant and allowing Ω to assume the value of spindle speed at any computational step allows the size of the discrete intervals to be approximately fixed. Even though this method is the most time-conserving, saving above 90 percent CT relative to the traditional case, it fails to produce accurate stability lobes with fall in radial immersion, fall in number of cutting edges and rise in spindle speed. Another proposed algorithm, which is preferred from the stand-point of accuracy, involved varying the factor

f according to equation (16) such that the approximation parameter r becomes a function of speed as given in equation (20). The two benefits of this method are redistribution of the biased accuracy of the traditional case in a way that favours the low speed domain and considerable savings in CT relative to the traditional case. With careful selection of the initial factor f_1 and final factor f_{l+1} , the method fixes all the problems of the other proposed method except for slight loss of the extreme CT savings.

REFERENCES

- [1] Smith, S. & Tlustý, J. (1993). Efficient Simulation Programs for Chatter in Milling. *CIRP Annals* 42 (1) 463–46.
- [2] Altintas, Y. & Budak, E. (1995). Analytical prediction of stability lobes in milling. *CIRP Annals—Manufacturing Technology* 44(1) 357–362.
- [3] Altintas, Y. (2001). Analytical prediction of three dimensional chatter stability in milling. *JSME International Journal, Series C: Mechanical Systems, Machine Elements and Manufacturing* 44 (3) 717–723.
- [4] Insperger, T. & Stepan, G. (2004). Updated semi-discretization method for periodic delay differential with discrete delay. *International Journal For Numerical Methods In Engineering* 61, 117–141.
- [5] Insperger, T.; Mann, B.P.; Stepan, G.; Bayly, P.V. (2003). Stability of up-milling and down-milling, part 1: alternative analytical methods. *International Journal of Machine Tools & Manufacture*, 43, 25–34.
- [6] Hartung, F.; Insperger, T.; Stepan, G.; & Turi, J. (2006). Approximate stability charts for milling processes using semi-discretization. *Applied Mathematics and Computation*, 174, 51–73.
- [7] Henninger, C. & Eberhard, P. (2008). Improving the computational efficiency and accuracy of the semi-discretization method for periodic delay-differential equations. *European Journal of Mechanics, A/Solids* 27 (6) 975–985.
- [8] Bobrenkov, O.A.; Khasawneh, F.A.; Butcher, E. A. & Mann, B.P. (2010). Analysis of milling dynamics for simultaneously engaged cutting teeth. *Journal of Sound and Vibration*, 329, 585–606.
- [9] Bayly, P.V.; Schmitz, T.L.; Stepan, G.; Mann, B.P.; Peters, D.A. & Insperger, T. (2002). Effects of radial immersion and cutting direction on chatter instability in end-milling. *Proceedings of IMECE'02 2002 ASME International Mechanical Engineering Conference & Exhibition* New Orleans, Louisiana, November 17-22.
- [10] Ding, Y.; Zhu, L.M.; Zhang, X.J. & Ding H. (2010). A full-discretization method for prediction of milling stability. *International Journal of Machine Tools and Manufacture*, 50, 502–509.
- [11] Insperger, T. (2010) Full-discretization and semi-discretization for milling stability prediction: Some comments. *International Journal of Machine Tools and Manufacture*, 50, 658–662.
- [12] Insperger, T. (2002). Stability Analysis of Periodic Delay-Differential Equations Modelling Machine Tool Chatter: PhD dissertation, *Budapest University of Technology and Economics*.
- [13] Faassen, R.P.H.; van de Wouw, N.; Oosterling, J.A.J. & Nijmeijer, H. (2003). Prediction of regenerative chatter by modelling and analysis of high-speed milling, *International Journal of Machine Tools & Manufacture* 43, 1437–1446.
- [14] Butcher, E. & Mann, B.P. Stability Analysis and Control of Linear Periodic Delayed Systems using Chebyshev and Temporal Finite Element Methods, Available from: http://mae.nmsu.edu/faculty/eab/bookchapter_final.pdf Accessed: 2014-05-09
- [15] Ding, Y.; Zhu, L.; Zhang, X. & Ding, H., (2010). Second-order full-discretization method for milling stability prediction, *International Journal of Machine Tools and Manufacture* 50, 926–932.
- [16] Quo, Q.; Sun, Y. & Jiang, Y. (2012). On the accurate calculation of milling stability limits using third-order full-discretization method, *International Journal of Machine Tools & Manufacture* 62, 61–66.
- [17] Yilong, L.; Dinghua, Z.; & Baohai W. (2012). An efficient full-discretization method for prediction of milling stability, *International Journal of Machine Tools & Manufacture* 63, 44–48.
- [18] Ozoegwu, C.G. (2014). Least squares approximated stability boundaries of milling process, *International Journal of Machine Tools and Manufacture*, 79, 24–30.
- [19] Insperger, T. & Stepan G. (2004). Stability improvements of robot control by periodic variation of the gain parameters. In *Proceedings of the 11th World Congress in Mechanism and Machine Science*, Tian Huang (ed.), Tianjin, China, 2003. China Machinery Press: China, 2004; 1816–1820.
- [20] Minorsky, N. (1942). Self-excited oscillations in dynamical systems possessing retarded actions. *Journal of Applied Mechanics*, 9, 65–71.
- [21] Stepan, G. & Kollar, L.E. (2000) Balancing with reflex delay. *Mathematical and Computer Modelling* 31, 199–205

- [22] Insperger, T. (2011). Stick balancing with reflex delay in case of parametric forcing. *Communication in Nonlinear Science and Numerical Simulation*, 16, 2160–2168.
- [23] Altintas, Y. (2000). *Manufacturing Automation: Metal Cutting Mechanics, Machine Tool Vibrations, and CNC Design*, Cambridge University Press, Cambridge.
- [24] Xiao, J.Z.; Cai, H.X.; Ye, D.; Ming, J.F. & You, L.X. (2012). Milling stability analysis with simultaneously considering the structural mode coupling effect and regenerative effect, *International Journal of Machine Tools & Manufacture*, 53, 127–140.
- [25] Ozoegwu, C.G. & Omenyi, S.N. (2014). Reducing computational requirement of stability analysis of milling by partial averaging. *Manufacturing Review*, DOI: 10.1051/mfreview/2014014.
- [26] Ozoegwu C.G.; Omenyi S.N. & Ofochebe S.M. (2015). Hyper-third order full-discretization methods in milling stability prediction, *International Journal of Machine Tools & Manufacture* 92, 1–9.